Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 14, Issue. 2, No. 2: 2023
ISSN : **1906-9685**

PROBABILISTIC INVENTORY MODEL WITH CONTINUOUS DEMAND IN FUZZY AND INTUITIONISTIC FUZZY ENVIRONMENTS

V Kamalakannan Department of Mathematics, Saveetha Engineering College (Autonomous) & Research Scholar, Pachaiyappa's College, Chennai, India.
G Uthra Assistant Professor, PG & Research, Department of Mathematics, Pachaiyappa's College, Chennai, India.

ABSTRACT:

In this paper, probabilistic Inventory model with continuous demand is considered in two situations, namely fuzzy and intuitionistic fuzzy environments We consider the holding cost (C_h), shortage cost (C_s), and set-up cost (C) of the units as trapezoidal fuzzy numbers and trapezoidal intuitionistic fuzzy numbers (TrFN and TrIFN). The optimal reorder point is found using the proposed method. The comparative results of both the environments are analysed using numerical examples.

Key words: Inventory models, Fuzzy sets, Intuitionistic Fuzzy Sets(IFS), TrFN, TrIFN

1.Introduction:

One of the most diverse areas of applied sciences is inventory system, which finds extensive application in a variety of industries such as computer sciences, management sciences, operations research, applied probability, production systems, and telecommunications. Referencing books and survey studies from over fifty years ago first discussed inventory system analysis. One of the earliest scholars to address inventory system analysis was Hadley and Whitin [1], who presented a technique for analyzing the mathematical model of inventory systems. Additionally, Balkhi and Benkherouf [2] created the production lot size inventory model, which allows for the variation in demand and production rates over time while maintaining a consistent rate of product deterioration. Both deterministic and probabilistic inventory models are possible as commodity demand might be either way. Vijayan and Kumaran [4], Abuo-El-Ata et al. [3], and Hadley and Whitin [1] all handled these cases.

A shortage in inventory systems, which might result from backorders, missed sales, or combination shortages, is tolerated by certain management. Numerous writers tackle inventory issues with different shortage scenarios, where the cost components are treated as sharp figures that don't accurately represent the actual inventory system. For instance, Fergany [5] has studied a limited probabilistic inventory model with changing order and shortage costs using the Lagrangian approach. Furthermore, Fergany and El-Saadani [6] reported on a limited probabilistic inventory model with continuous distributions and variable holding costs. Using the Lagrangian technique, Fergany and El-Wakeel [7] presented a number of continuous distribution models for restricted probabilistic lost sales inventory models with variable order costs under holding cost constraints in 2006. Using the Lagrangian approach, El-Wakeel [8] recently calculated a constrained backorders inventory system with variable order costs and lead times and uniformly distributed demand.

Occasionally, the cost components are regarded as fuzzy values because, in practice, different physical or chemical properties may have an impact on the cost components. As a result, it becomes challenging to measure the precise values of cost characteristics, such as the precise amount of order, holding, and particularly shortage cost. As a result, when regulating the inventory system, some flexibility in the cost parameter values may be allowed in order to deal with the uncertainties that inevitably occur in real-world scenarios. To some extent, the fuzzy set theory fits our needs for such contradictions. The fuzzy set theory was first presented by Zadeh in 1965, and it focused on the desire to account for uncertainty in a nonstochastic manner as opposed to the existence of random variables Syed and Aziz [9] used the signed distance approach to investigate the fuzzy inventory model without shortages. The inventory model with backorders was handled by Kazemi et al. [10] using fuzzy parameters and decision variablesFergany and Gawdt's [11] continuous review inventory model with mixture shortage under constraint incorporating crashing cost was examined. Kumar and Rajput [12] develop a fuzzy inventory model for decaying products with time-dependent demand and partial backlog. An inventory model for continuous review in a fuzzy environment with no backorder for degrading commodities was recently presented by Patel et al. [13].

2. Definition and Preliminaries

Definition 2.1:

The set of ordered pairs forms a fuzzy set X in the given set$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where the membership function is referred to as $\mu_{\tilde{A}}: X \to [0,1]$.

Definition 2.2.

A TrFN $\tilde{A} = (r_1, r_2, r_3, r_4)$ is displayed using the membership function $\mu_{\tilde{A}}$ with

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x - r_1}{r_2 - r_1}, \text{when } r_1 \le x \le r_2 \\ 1 \qquad\qquad\quad, \text{when } r_2 \le x \le r_3 \\ \mathrm{R(x)} = \dfrac{r_4 - x}{r_4 - r_3}, \text{when } r_3 \le x \le r_4 \\ 0 \qquad\qquad\quad, otherwise \end{cases}$$

Definition 2.3.

Suppose $\tilde{A} = (l_1, l_2, l_3, l_4)$ and $\tilde{B} = (m_1, m_2, m_3, m_4)$ are two TrFN, then the arithmetical operations are given as:

(i) $\tilde{A} \oplus \tilde{B} = (l_1 + m_1, l_2 + m_2, l_3 + m_3, l_4 + m_4)$

(ii) $\tilde{A} \otimes \tilde{B} = (l_1 m_1, l_2 m_2, l_3 m_3, l_4 m_4)$

(iii) $\tilde{A}\, \Theta \tilde{B} = (l_1 - m_4, l_2 - m_3, l_3 - m_2, l_4 - m_1)$

(iv) $\tilde{A} \Phi \tilde{B} = \left(\frac{l_1}{m_4}, \frac{l_2}{m_3}, \frac{l_3}{m_2}, \frac{l_4}{m_1}\right), provided\ m_1, m_2, m_3, m_4 \neq 0$

Definition 2.4.

A TrIFN $\tilde{A}$ is with membership function $\mu_{\tilde{A}}(x)$ and non-membership function $I_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - r_1}{r_2 - r_1}, \text{when } r_1 \le x \le r_2 \\ 1\ \ , \text{when } r_2 \le x \le r_3 \\ \frac{r_4 - x}{r_4 - r_3}, \text{when } r_3 \le x \le r_4 \\ 0\ \ , otherwise \end{cases} \text{ and } I_{\tilde{A}}(x) = \begin{cases} \frac{r_2 - x}{r_2 - r_1{}'}, \text{when } r_1{}' \le x \le r_2 \\ 0\ \ , \text{when } r_2 \le x \le r_3 \\ \frac{x - r_3}{r_4{}' - r_3}, \text{when } r_3 \le x \le r_4{}' \\ 1, \quad \text{otherwise} \end{cases}$$

Where$r_1{}' < r_1 < r_2 < r_3 < r_4 < r_4'$.

Definition 2.5.

From the output of the merged fuzzy set, a single number is taken out in the defuzzification stage. It is employed to provide a distinct output from fuzzy inference's results. An algorithm that makes decisions chooses the best crisp value from a set of fuzzy values is consequently responsible for defuzzification. Center of gravity (COG), mean of maximum (MOM), and center average techniques are just a few of the several ways defuzzification can be applied. While the MOM technique shows the point at which a curve reaches equilibrium, the COG approach yields the value of the area under the curve's center. We apply the following defuzzification measure for TrFN and TrIFN.

For TrFN $\tilde{A} = (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4)$

$$R(\tilde{A}) = \frac{\vartheta_1 + \vartheta_2 + \vartheta_3 + \omega_4}{4}$$

For TrIFN $\tilde{B} = (\vartheta_1', \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_4')$

$$R(\widetilde{B}) = \frac{\vartheta_1' + \vartheta_1 + \vartheta_2 + \vartheta_3 + \vartheta_4 + \vartheta_4'}{6}$$

3.Model 1: Continuous Demand with Discrete Replenishment

This model is similar to Instantaneous demand with shortages, except that the cost equations for continuous demand and discrete replenishment need to be developed in different manner. The following circumstance may develop as a result of the same reasoning employed in the derivation:

Case I: Demand is lower than supply.

Only the carrying costs would be incurred in this situation. This cost is computed using the scenario depicted in Figure 1.

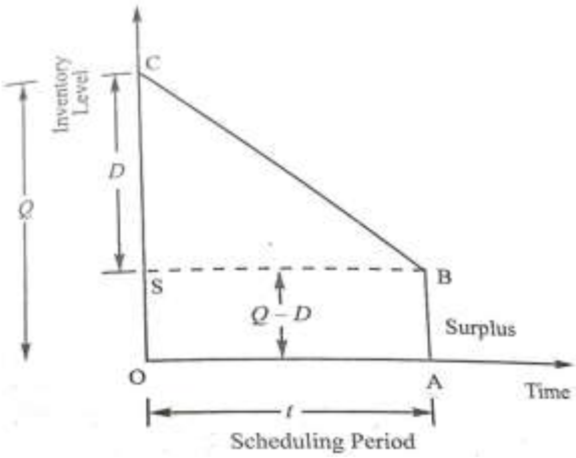

Figure 1.

Carrying cost $= C_h *$ Inventory area of OABC

$$= C_h * \frac{1}{2}[AB + OC]SB(\text{area of trapezium})$$

$$= C_h * \frac{1}{2}[Q - D + Q] * t$$

$$= C_h * \frac{t}{2}[2Q - D]$$

$$\text{Expected carrying cost} = \mathrm{C_h} * \frac{t}{2}\sum_{D=0}^{Q}[2Q - D]f(D); D \leq Q$$

Case II: Demand is greater than supply.

Only the cost of the shortage would be borne in this scenario. The solution shown in Figure 2 is used to calculate this cost.

$$Inventory\ area\ of\,\Delta OAS = \frac{1}{2}\{OS * OA\}$$

$$= \frac{1}{2}\left\{Q * \frac{Q.t}{D}\right\} = \frac{1}{2}\frac{Q^2 t}{D}$$

From property of similar triangles ($\Delta SEC\ and\ \Delta OAS$),

$$\frac{SO}{SE} = \frac{AO}{EC}\ or\ \frac{Q}{D} = \frac{OA}{t}$$

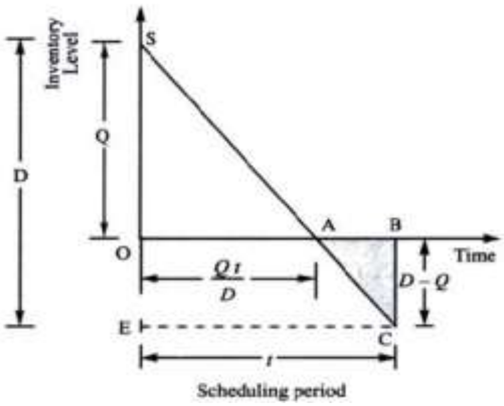

Figure 2.

In Fig 2 the shortage is shown by ΔABC. Therefore

Area of $\Delta ABC = \frac{1}{2}(AB * BC) = \frac{1}{2}(EC - OA) * BC$

$$= \frac{1}{2} * \left(t - \frac{Qt}{D}\right) * (D - Q) = \frac{1}{2D} * t * (D - Q)^2$$

The following provides the expected shortage cost

$$\sum_{D=Q+1}^{\infty} \left[C_h \frac{Q^2}{2D} * t + \frac{C_s}{2D} * t * (D - Q)^2\right] f(D), where\ D > Q \ldots\ldots\ldots\ldots (1)$$

The equation that determines the total expected c

$$TEC(Q) = C_h t \sum_{D=0}^{Q} \left[Q - \frac{D}{2}\right] f(D) + \sum_{D=Q+1}^{\infty} \left[C_h \frac{Q^2}{2D} * t + \frac{C_s}{2D} * (D - Q)^2 * t\right] f(D)$$

…………(2)

To determine the optimal value $Q^*$of Q in order to minimize $TEC(Q)$, The condition that follows must be true

$$\Delta TEC(Q^* - 1) < 0 < \Delta TEC(Q^*)$$

We can deduce from difference equations principles that

$$\Delta TEC(Q) = TEC(Q + 1) - TEC(Q)$$

Consequently, substituting Q for Q+1 in eqn(2), we obtain

$$TEC(Q + 1) = t * \left(C_h \sum_{D=0}^{Q+1} \left[Q + 1 - \frac{D}{2}\right] f(D) + C_h \sum_{D=Q+2}^{\infty} \frac{(Q + 1)^2}{2D} f(D) + C_s \sum_{D=Q+2}^{\infty} \frac{(D - Q - 1)^2}{2D} * f(D)\right)$$

…………(3)

From eqns (2)&(3), we have:

$$\Delta TEC(Q) = TEC(Q + 1) - TEC(Q)$$

$$= t * \left[(C_h + C_s) * \left\{F(Q) + \left(Q + \frac{1}{2}\right) \sum_{D=Q+1}^{\infty} \frac{f(D)}{D}\right\} - C_s\right] \ldots\ldots\ldots\ldots\ldots (4)$$

$$where, F(Q) = \sum_{D=0}^{Q} f(D)$$

$$L(Q) = F(Q) + \left(Q + \frac{1}{2}\right) \sum_{D=Q+1}^{\infty} \frac{f(D)}{D}$$

Then Eq(4) becomes

$$\Delta TEC(Q) = TEC(Q+1) - TEC(Q) = t * [(C_h + C_s)L(Q) - C_s] \text{ ....(5)}$$

Similarly, replacing Q with (Q-1) in Eq(5), we have:

$$\Delta TEC(Q-1) = TEC(Q) - TEC(Q-1) = t * [(C_h + C_s)L(Q-1) - C_s] \text{ ....(6)}$$

But $\Delta TEC(Q) > 0$ and $\Delta TEC(Q-1) < 0$ for minimum $TEC(Q)$; Consider the following Q* value:

$(C_h + C_s)L(Q^*) - C_s \geq 0$

$(C_h + C_s)L(Q^* - 1) - C_s \leq 0$ ....(7)

For any $Q^* + 1 > Q^*$ and $Q^* - 1 < Q^*$ inequalities, eq(7) holds since $L(Q)$ is non-decreasing for increasing Q. Therefore, by changing the terms in equation (7), we obtain

$$L(Q^* - 1) \leq \frac{C_s}{C_h + C_s} \leq L(Q^*)$$

Model 1(b): Continuous Demand, Continuous Replenishment

Given a continuous density function P(D) representing the demand for D units of an item, then

$$TEC(Q) = C_h.t \int_0^Q \left(Q - \frac{D}{2}\right) P(D)dD + t * \int_Q^{\infty} \left[C_h \frac{Q^2}{2D} + \frac{C_s}{2D}(D-Q)^2\right] P(D)$$

-------------------(8)

Eq (8) must first be differentiated with regard to "Q" in order to find the ideal order size Q^* that will minimise TEC(Q).

To determine the optimal order size Q^* so as to minimize TEC(Q), first differentiate Eq(8) with respect to “Q”

Afterwards, after simplifying, equal to zero gives us:

$$\frac{d(TEC)}{dQ} = t * (C_h + C_s) * [F(Q) + G(Q)] - C_s * t$$

$$= t * (C_h + C_s)\, L(Q) - C_s * t$$

$$where, F(Q) = \int_0^Q P(D)dD;\ G(Q) = Q \int_Q^Q \frac{P(D)}{D} dD;\ \ L(Q) = F(Q) + G(Q)$$

The TEC(Q) will be at its absolute minimum at $Q = Q^*$if

$$\frac{d(TEC)}{dQ} = 0$$

$$t * (C_h + C_s)\, L(Q^*) - C_s * t = 0$$

$$L(Q^*) = \frac{C_s}{(C_h + C_s)}$$

Now

$$\frac{d^2(TEC)}{dQ^2} = t * (C_h + C_s) \int_Q^{\infty} \frac{P(D)}{D}\, dD > 0$$

Numerical Example 3.1: Consider the single – period model whose holding cost varies between 100 t0 200 and shortage cost ranges from 400 to 500. The density function of demand is given by

D	:0	1	2	3	4	5
P(D)	: 0.10	0.20	0.25	0.20	0.15	0.10

Find the Optimum order quantity.

Solution:

D	0	1	2	3	4	5
Cumulative function	0.1	0.3	0.55	0.75	0.90	1

Assume that the costs associated with holding, shortage, and purchase are all Trapezoidal Fuzzy numbers.

For TrFN, $\widetilde{C_h} = (100,140,180,200)$

$$\widetilde{C_S} = (400,430,470,500)$$

$$\widetilde{C_h} + \widetilde{C_s} = (500,570,650,700)$$

$$\frac{\widetilde{C_s}}{\widetilde{C_h} + \widetilde{C_s}} = \left(\frac{4}{7}, \frac{43}{65}, \frac{47}{57}, \frac{5}{5}\right) = (0.5714,0.6615,0.8246,1) = 0.7644$$

For TrIFN, $\widetilde{C_h} = (100,120,140,180,190,200)$

$$\widetilde{C_S} = (400,415,430,470,485,500)$$

$$\widetilde{C_h} + \widetilde{C_s} = (500,535,570,650,675,700)$$

$$\frac{\widetilde{C_s}}{\widetilde{C_h} + \widetilde{C_s}} = \left(\frac{4}{7}, \frac{415}{675}, \frac{43}{65}, \frac{47}{57}, \frac{485}{535}, \frac{5}{5}\right) = (0.5714,0.6148,0.6615,0.8246,0.9065,1) = 0.7631$$

Therefore, the optimum order quantity is 4.

4.Model 2 development:

This model is the same as model I, with the exception that purchasing or producing products within a specific time frame is linked to a fixed set-up cost, denoted by K. At the start of the period, let I be the inventory level. This suggests that a size $Q - I$ item purchase will be placed in order to bring the item's available inventory up to Q. As a result, the expected cost will be :

$$TEC'(Q) = K + C \ * \ (Q - I) + C_h \int_{D=0}^{Q} (Q - D) * P(D)dD + C_s \int_{D=Q}^{\infty} (D - Q) * P(D)dD$$

$$= K + TEC(Q)\ldots\ldots\ldots\ldots\ldots(9)$$

The following determines the ideal value of Q, let's say Q^*, to minimize TEC(Q):

$$F(Q^* - 1) \leq \frac{C_s - C}{(C_h + C_s)} \leq F(Q^*)$$

$$where\ F(Q) = \int_{D=0}^{Q} P(D)dD$$

Q^* will also minimise $TEC'(Q)$because "K" is constant and the least value of $TEC'(Q)$ must likewise be determined by the same criterion as stated in eq (9).

The maximum stock level is represented by the variable S, and the reordered level is indicated by the variable s. Let's introduce these new control variables. In other words, an order is issued to raise the stock items' stock to S when the stock level falls below that threshold. Hence, the relationship determines the value of s and the value of $S = Q^*$

$$TEC(s) = TEC'(S) = K + TEC(S); s < S$$

Assuming that I represents the original inventory before to the period's start, the following three scenarios can be examined in order to determine the order size that will raise the on-hand inventory to Q^*:

$$(a)\ I < s, (b) s \leq I \leq S, (c) I > S$$

Case(a):

The estimated cost is $TEC(I)$ if we have " I" units of inventory at the beginning of the period and do not purchase or create any more. However, $TEC'(Q^*)$ includes the setup cost if we plan to purchase extra $(Q - I)$ units to raise the inventory level to Q^*.So, the ordering requirement for all $I \ < \ s$ is:

$$\underset{Q>I}{\text{Min}}\{TEC'(\, Q^*)\} = TEC'(S) < TEC(I)$$

That is, orders for $(Q - I)$ inventory units may be placed whenever the inventory level exceeds $S = Q^*$

Case(b):

Here, the criterion determines the order size if $I \ < \ Q$

$$TEC(I) \leq \underset{Q>I}{\text{Min}}\{TEC'(\, Q) = TEC'(S)$$

This suggests that there is no ordering that is less costly than ordering. Thus, $Q^* = I$.

Case(c):

If Q is more than I, the projected cost of an order up to Q will be higher than the expected cost of the order as a whole., i.e.

$$TEC(Q) \geq TEC(I)$$

As a result, it is preferable not to make an order for item procurement, and then Q*=I.

Numerical Example 4.1:

Determine the best order policy if the carrying cost is between Rs 0.5 and Rs 1 per unit and the shortage cost is between Rs 5 and Rs 10 per unit, given that the demand for a given product has a rectangular distribution between 100 and 200.Ten units were available at the start of the period; the cost of setup was Rs. 25, and the price of each unit ranged from Rs. 2 to Rs. 5.

Solution:

Trapezoidal fuzzy numbers are taken into consideration for holding, shortage, and purchase costs.

$$P(D) = \frac{1}{200-100} = \frac{1}{100}$$

For TrFN

$$\widetilde{C_h} = (0.5,0.7,0.8,1)$$

$$\widetilde{C_s} = (5,7,8,10)$$

$$\tilde{C} = (2,3,4,5)$$

$$\int_{D=0}^{Q} \frac{1}{100} dD = \frac{\widetilde{C_s} - \tilde{C}}{\widetilde{C_h} + \widetilde{C_s}}$$

$$\frac{Q}{100} = \frac{\widetilde{C_s} - \tilde{C}}{\widetilde{C_h} + \widetilde{C_s}} = \frac{(0,3,5,8)}{(5.5,7.7,8.8,11)}$$

$$= (0,0.3409,0.6494,1.4546) = 0.6112$$

$$\frac{Q}{100} = 0.6112$$

$$Q = 61\ Units$$

Setting $Q = I = 10$yields the value of TEC(I).

Thus, it is necessary to arrange $Q^* - I = 61 - 10 = 51$ units in the ideal order.

For TrIFN,

$$\widetilde{C_h} = (0.5,0.6,0.7,0.8,0.9,1)$$

$$\widetilde{C_s} = (5,6,7,8,9,10)$$

$$\tilde{C} = (2,2.5,3,4,4.5,5)$$

$$\int_{D=0}^{Q} \frac{1}{100} dD = \frac{\widetilde{C_s} - \tilde{C}}{\widetilde{C_h} + \widetilde{C_s}}$$

$$\frac{Q}{100} = \frac{\widetilde{C_s} - \tilde{C}}{\widetilde{C_h} + \widetilde{C_s}} = \frac{(0,1.5,3,5,6.5,8)}{(5.5,6.6,7.7,8.8,9.9,11)}$$

$$= (0,0.1515,0.3409,0.6494,0.9849,1.4546) = 0.5969$$

$$\frac{Q}{100} = 0.5969$$

$$Q = 60\ Units$$

Setting $Q = I = 10$yields the value of TEC(I).

Thus, it is necessary to arrange $Q^* - I = 61 - 10 = 51$ units in the ideal order.

5.Conclusion:

In this paper, probabilistic inventory models are considered whose cost values are not deterministic. The upper and lower limit values of the costs are known and hence the costs involved are considered as TrFN and TrIFN. The economic order quantity is found using traditional method and by converting the fuzzy values into crisp values. This approach would be more appropriate in dealing with probabilistic inventory models with imprecise and vague information.

References:

[1] G. Hadley and T. M. Whitin, Analysis of Inventory System, Prentice Hall, Englewood Cliffs, NJ, USA, 1963.

[2] Z. T. Balkhi and L. Benkherouf, "A production lot size inventory model for deteriorating items and arbitrary production and demand rates," European Journal of Operational Research, vol.92, no. 2, pp. 302–309, 1996.

[3] M. O. Abuo-El-Ata, H. A. Fergany, and M. F. El-Wakeel, "Probabilistic multi-item inventory model with varying order cost under two restrictions: a geometric programming approach," International Journal of Production

[4] T. Vijayan and M. Kumaran, "Inventory models with a mixture of backorders and lost sales under fuzzy cost," European Journal of Operational Research, vol. 189, no. 1, pp. 105–119, 2008.

[5] H. A. Fergany, Inventory models with demand-dependent units cost [Ph.D. dissertation], Faculty of Science, Tanta University,1999.

[6] H. A. Fergany and M. E. El-Saadani, "Constrained probabilistic inventory model with continuous distributions and varying holding cost," International Journal of Applied Mathematics, vol. 17, pp. 53–67, 2005.

[7]H. A. Fergany and M. F. El-Wakeel, "Constrained probabilistic lost sales inventory system with continuous distributions and varying order cost," Journal of Association for the Advancement of Modelling and Simulation Techniques in Enterprises, vol. 27, pp. 3–4, 2006.

[8] M. F. El-Wakeel, "Constrained backorders inventory system with varying order cost: lead time demand uniformly distributed," Journal of King Saud University—Science, vol. 24, no. 3, pp. 285–288, 2012.

[9] J. K. Syed and L. A. Aziz, "Fuzzy inventory model without shortages using signed distance method," International Journal of Applied Mathematics and Information Sciences, vol. 1, no. 2, pp.203–209, 2007.

[10] N. Kazemi, E. Ehsani, and M. Y. Jaber, "An inventory model with backorders with fuzzy parameters and decision variables," International Journal of Approximate Reasoning, vol. 51, no. 8,pp. 964–972, 2010.

[11] H. A. Fergany and O. A. Gawdt, "Continuous review inventory model with mixture shortage under constraint involving crashing cost based on probabilistic triangular fuzzy numbers," The Online Journal on Mathematics and Statistics, vol. 2, no. 1, pp. 42–48, 2011.

[12] S. Kumar and U. S. Rajput, "Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging," International Journal of Applied Mathematics, vol. 6, no.3, pp. 496–509, 2015.

[13] P. D. Patel, A. S. Gor, and P. Bhathawala, "Continuous review inventory model under fuzzy environment without backorder for deteriorating items," International Journal of Applied Research, vol. 2, no. 3, pp. 682–686, 2016.